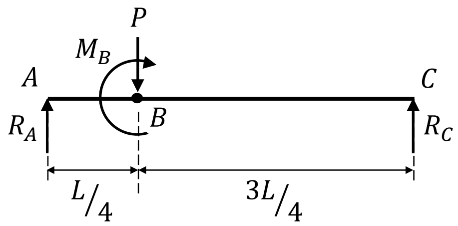
2016-2017 MM2MS3 Exam Solutions

1.

(a)

Placing the applied load and moment on the beam and adding the reaction loads at the support positions:



[1 mark]

Where, it can be seen from the figure in the question that:

$$M_B = \frac{PL}{4} \qquad (1)$$

[1 mark]

Vertical equilibrium of the beam:

$$R_A + R_C = P \qquad (2)$$

[1 mark]

Taking moments about position C:

$$\frac{3PL}{4} = R_A L + M_B$$

$$\therefore R_A = \frac{3P}{4} - \frac{M_B}{L}$$

[1 mark]

Substituting (1) into this:

$$\boldsymbol{R_A = \frac{P}{2} = 3.5\ \mathrm{kN}} \qquad (3)$$

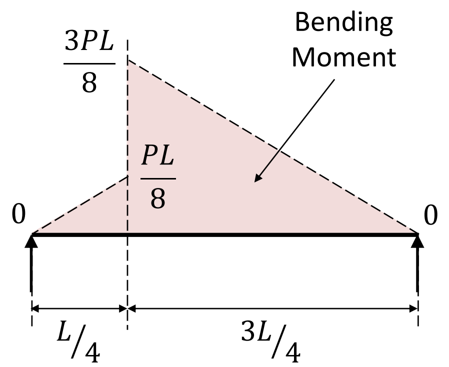
[2 marks]

Substituting this into (2) gives:

$$\boldsymbol{R_C = \frac{P}{2} = 3.5\ \mathrm{kN}}$$

[2 marks]

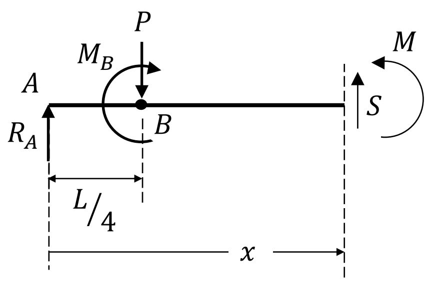
Bending Moment distribution:



[2 marks]

(b)

Taking the origin at the left-hand end of the beam, sectioning after the last discontinuity and drawing a free body diagram of the left-hand side of the section:



[3 marks]

Taking moments about the section position:

$$M + P\left\langle x - \frac{L}{4}\right\rangle = R_A x + M_B \left\langle x - \frac{L}{4}\right\rangle^0$$

$$\therefore M = R_A x + M_B \left\langle x - \frac{L}{4}\right\rangle^0 - P\left\langle x - \frac{L}{4}\right\rangle$$

[2 marks]

Substituting this into the main deflections of beams equation ($EI\frac{d^2y}{dx^2} = M$):

$$EI\frac{d^2y}{dx^2} = R_A x + M_B \langle x - \frac{L}{4}\rangle^0 - P\langle x - \frac{L}{4}\rangle$$

[1 mark]

Integrating with respect to x:

$$EI\frac{dy}{dx} = \frac{R_A x^2}{2} + M_B \langle x - \frac{L}{4}\rangle - \frac{P\langle x - \frac{L}{4}\rangle^2}{2} + A$$

[1 mark]

Integrating with respect to x again:

$$EIy = \frac{R_A x^3}{6} + \frac{M_B\langle x - \frac{L}{4}\rangle^2}{2} - \frac{P\langle x - \frac{L}{4}\rangle^3}{6} + Ax + B \qquad (4)$$

[1 mark]

Boundary conditions:

(BC1) At $x = 0, y = 0$, therefore from (4):

$$B = 0$$

[1 mark]

(BC2) At $x = L, y = 0$, therefore from (4):

$$0 = \frac{R_A L^3}{6} + \frac{9M_B L^2}{32} - \frac{27PL^3}{384} + AL$$

Substituting (1) and (3) into this and rearranging:

$$A = -\frac{PL^2}{12} \qquad (5)$$

[1 mark]

From (4), at $x = \frac{L}{4}$ (point B):

$$EIy = \frac{R_A L^3}{384} + \frac{AL}{4}$$

[2 marks]

Substituting (3) and (5) into this:

$$y = -\frac{5PL^3}{256EI}$$

[1 mark]

Substituting values of P, L, E & I:

$$\therefore \boldsymbol{y = -2.083\ \mathbf{mm}}$$

(i.e. downward deflection)

[2 marks]

2.

(a)

Calculation of position of Neutral Axis:

$$\sigma_b = \frac{M_P Q}{I_P} - \frac{M_Q P}{I_Q}$$

At the Neutral Axis, $\sigma_b = 0$, therefore,

$$\frac{M_P Q}{I_P} - \frac{M_Q P}{I_Q} = 0$$

$$\therefore \frac{M_P Q}{I_P} = \frac{M_Q P}{I_Q}$$

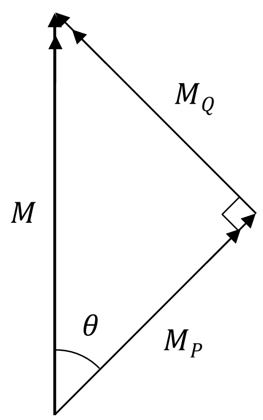
$$\therefore \frac{Q}{P} = \frac{M_Q I_P}{M_P I_Q}$$

Therefore, α, the angle between the neutral axis and the principal axes can be defined as,

$$\alpha = \tan^{-1}\left(\frac{Q}{P}\right) = \tan^{-1}\left(\frac{M_Q I_P}{M_P I_Q}\right) \qquad (1)$$

[1 mark]

Resolve applied bending moment onto Principal Axes



Therefore,

$$M_P = M cos\theta = 125 cos 45 = 88.39 Nm = 88{,}390 \text{ Nmm}$$

and,

$$M_Q = M sin\theta = 125 sin45 == 88.39Nm = 88{,}390 \text{ Nmm}$$

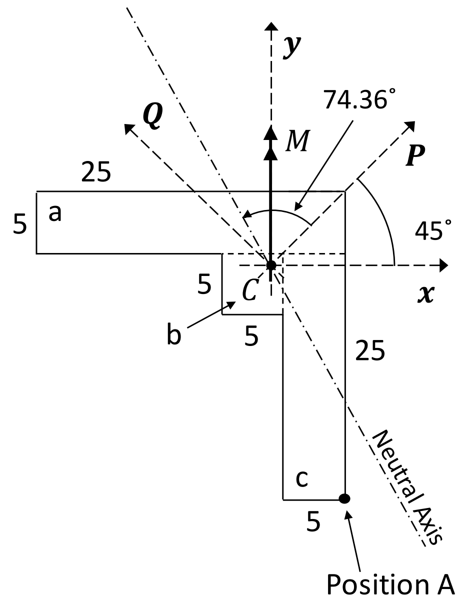
[1 mark]

Substituting these values, and the Principal Second Moments of Area (I_P and I_Q – given in the question) into equation (1) gives:

$$\alpha = \tan^{-1}\left(\frac{88{,}390 \times 19{,}270.83}{88{,}390 \times 5{,}395.83}\right) = \tan^{-1}(3.571) = 74.36°$$

[1 mark]

Therefore, the neutral axis is at 74.36° (anti-clockwise) from the principal axes as shown below,



The neutral axis is therefore at (45° + 74.36° =) 119.36° anti-clockwise from the x-axis.

[2 mark]

(b)

Stresses at position A:

As above,

$$\sigma_b = \frac{M_P Q}{I_P} - \frac{M_Q P}{I_Q}$$

[1 mark]

M_P, M_Q, I_P and I_Q are known, therefore, the co-ordinates of point A on the $P - Q$ axes are required. The translation from the $x - y$ axes to the $P - Q$ axes for this situation is as follows:

$$P = P_x + P_y = xsin45 + ycos45$$

and,

$$Q = -Q_x + Q_y = -xcos45 + ysin45$$

[2 marks]

At position A, $x = 8 \text{ mm}$ and $y = -17 \text{ mm}$. Therefore,

$$P = 8sin45 - 17cos45 = -6.36 \text{ mm}$$

and,

$$Q = -8cos45 - 17sin45 = -17.68 \text{ mm}$$

[3 marks]

Therefore, these $P - Q$ co-ordinates for position A can now be substituted into the equation for bending stress to give:

$$\sigma_{bA} = \frac{M_P Q}{I_P} - \frac{M_Q P}{I_Q} = -81.0933 + 104.1842 = \frac{88{,}390 \times -17.68}{19{,}270.83} - \frac{88{,}390 \times -6.36}{5{,}395.83}$$

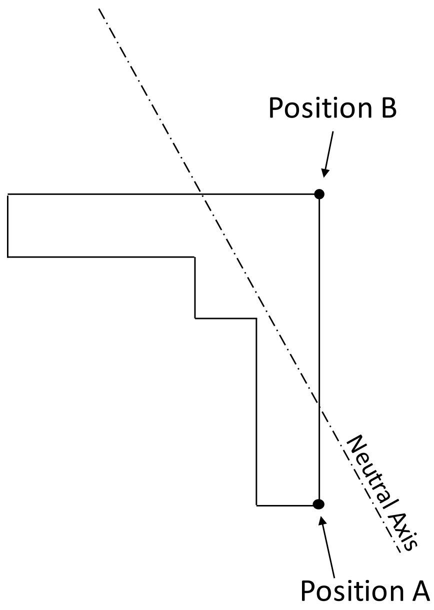
$$\therefore \boldsymbol{\sigma_{bA} = 23.09 \text{ MPa}}$$

[3 marks]

(c)

Maximum Compressive Stress:

It is thought that the maximum compressive stress will be at position B as indicated in the figure below,



[3 marks]

At position B, $x = 8$ mm and $y = 8$ mm. Therefore,

$$P = 8sin45 + 8cos45 = 11.31 \text{ mm}$$

and,

$$Q = -8cos45 + 8sin45 = 0 \text{ mm}$$

[3 marks]

Therefore, these $P - Q$ co-ordinates for position B can now be substituted into the equation for bending stress to give:

$$\sigma_{bA} = \frac{M_P Q}{I_P} - \frac{M_Q P}{I_Q} = \frac{88{,}390 \times 0}{19{,}270.83} - \frac{88{,}390 \times 11.31}{5{,}395.83}$$

$$\therefore \boldsymbol{\sigma_{bB} = -186.4 \text{ MPa}}$$

[3 marks]

3.

(a)

Applying Lame's equations for the inner cylinder (1):

$$\sigma_{r1} = A_1 - \frac{B_1}{r^2}$$

$$\sigma_{\theta 1} = A_1 + \frac{B_1}{r^2}$$

[1 mark]

at $r = 50 \text{ mm}, \sigma_r = 0$

$$0 = A_1 - \frac{B_1}{2500}$$

$$B_1 = 2500A_1 \qquad (1)$$

at $r = 90 \text{ mm}, \sigma_r = -p$ (interface pressure)

$$-p = A_1 - \frac{B_1}{8100}$$

If we substitute equation (1) into this, we can remove B_1 from the expression:

$$-p = A_1 - \frac{2500}{8100}A_1$$

[1 mark]

And rearrange to give:

$$\boldsymbol{A_1 = -1.45p}$$

[2 marks]

And therefore:

$$B_1 = 2500 \times (-1.45p)$$

$$\therefore \boldsymbol{B_1 = -3616p}$$

[2 marks]

Lame's equations for the outer cylinder (2):

$$\sigma_{r2} = A_2 - \frac{B_2}{r^2}$$

and,

$$\sigma_{\theta 2} = A_2 + \frac{B_2}{r^2}$$

At $r = 130$ mm, $\sigma_r = 0$

$$0 = A_2 - \frac{B_2}{16900}$$

$$\therefore B_2 = 16900A_2 \qquad (2)$$

At $r = 90$ mm, $\sigma_r = -p$ (interface pressure)

$$-p = A_2 - \frac{B_2}{8100}$$

Substitute in equation (2) in order to remove B_2 from the expression:

$$-p = A_2 - \frac{16900}{8100}A_2$$

[1 mark]

Rearranging:

$$\boldsymbol{A_2 = 0.92p}$$

[2 marks]

And therefore:

$$B_2 = 16900 \times (0.92p)$$

$$\therefore \boldsymbol{B_2 = 15555.7p}$$

[2 marks]

(b)

The general expressions for the inner cylinder (1) are:

$$\sigma_{r1} = -1.45p\left(1 - \frac{2500}{r^2}\right)$$

$$\sigma_{\theta 1} = -1.45p\left(1 + \frac{2500}{r^2}\right)$$

And the general expressions for the outer cylinder (2) are:

$$\sigma_{r2} = 0.92p\left(1 - \frac{16900}{r^2}\right)$$

$$\sigma_{\theta 2} = 0.92p\left(1 + \frac{16900}{r^2}\right)$$

However, at this stage p is still unknown so we cannot solve for the stresses.

We now need to consider compatibility:

$$i_1 + i_2 = i = 0.09 \text{ mm}$$

[1 mark]

Recalling:

$$\varepsilon_\theta = \frac{u}{r} = \frac{1}{E}\left(\sigma_\theta - \nu(\sigma_r + \sigma_z)\right)$$

As $\sigma_z = 0$, this reduces to:

$$\varepsilon_\theta = \frac{u}{r} = \frac{1}{E}(\sigma_\theta - \nu\sigma_r)$$

[1 mark]

At the outside of cylinder 1, $r = 40$ mm,

$$\frac{-i_1}{90} = \frac{1}{210000}(\sigma_\theta - \nu\sigma_r) = \frac{1}{210000}(-1.45p)\left(1 + \frac{2500}{8100} - 0.3\left(1 - \frac{2500}{8100}\right)\right)$$

$$i_1 = 6.83 \times 10^{-4} p$$

[1 mark]

At the inside of cylinder 2, $r = 90$ mm,

$$\frac{+i_2}{90} = \frac{1}{210000}(\sigma_\theta - \nu\sigma_r) = \frac{1}{210000}0.92p\left(1 + \frac{16900}{8100} - \nu\left(1 - \frac{16900}{8100}\right)\right)$$

$$i_2 = 1.35 \times 10^{-3}p$$

[1 mark]

As:

$$i_1 + i_2 = i = 0.09 \text{ mm}$$

$$\therefore 6.83 \times 10^{-4} + 1.35 \times 10^{-3} = 0.09$$

$$2.03 \times 10^{-3}p = 0.09$$

[1 mark]

we can now rearrange to determine a value for p:

$$p = \frac{0.09}{2.03 \times 10^{-3}} = 44.36 \text{ MPa}$$

[2 mark]

Now we can substitute the value for *p* into to get the following equations for the variation in stresses in the inner cylinder (1):

$$\sigma_{r1} = -64.17\left(1 - \frac{2500}{r^2}\right)$$

$$\sigma_{\theta 1} = -64.17\left(1 + \frac{2500}{r^2}\right)$$

[1 mark]

While substituting for p to give the following for the outer cylinder (2):

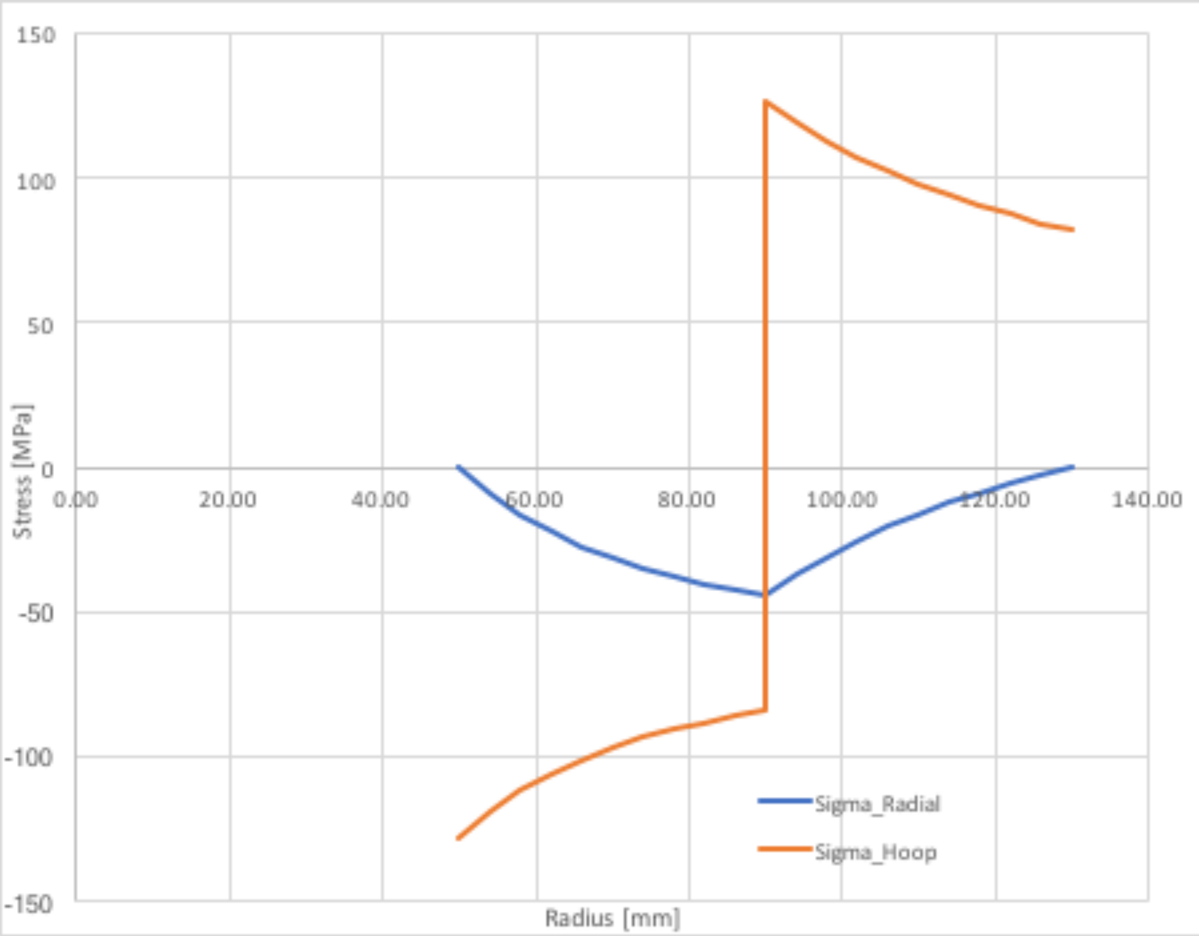
$$\sigma_{r2} = 40.83\left(1 - \frac{16900}{r^2}\right)$$

$$\sigma_{\theta 2} = 40.83\left(1 + \frac{16900}{r^2}\right)$$

[1 mark]

	r	Sigma_Radial	Sigma_Hoop
Cyl1	**50.00**	**0.00**	**-128.33**
	54.00	-9.15	-119.18
	58.00	-16.48	-111.85
	62.00	-22.43	-105.90
	66.00	-27.34	-100.99
	70.00	**-31.43**	**-96.90**
	74.00	-34.87	-93.46
	78.00	-37.80	-90.53
	82.00	-40.31	-88.02
	86.00	-42.48	-85.86
	90.00	**-44.36**	**-83.97**
Cyl2	**90.00**	**-44.36**	**126.03**
	94.00	-37.27	118.93
	98.00	-31.02	112.69
	102.00	-25.50	107.16
	106.00	-20.58	102.25
	110.00	**-16.20**	**97.87**
	114.00	-12.27	93.93
	118.00	-8.73	90.39
	122.00	-5.53	87.20
	126.00	-2.63	84.30
	130.00	**0.00**	**81.67**

[2 marks]


150
100
50
0
-50
-100
-150
Stress [MPa]
0.00
20.00
40.00
60.00
80.00
100.00
120.00
140.00
Radius [mm]
Sigma_Radial
Sigma_Hoop

[3 marks]

4.

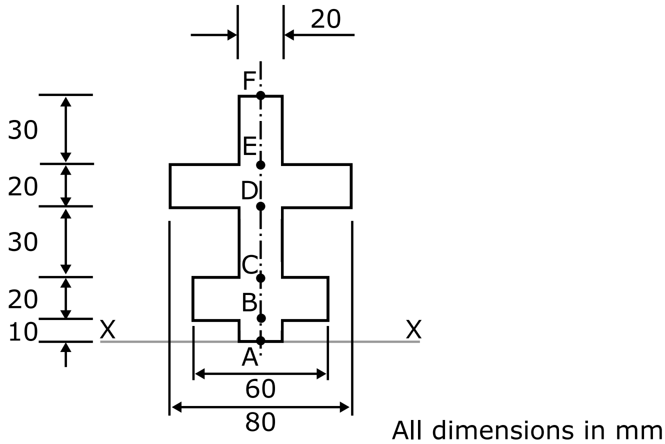
The section shown in Figure Q4 carries a shear force, $S = 48\ \text{kN}$ down the vertical centre line.

(a)

Using first moment of area:

$$\bar{y} = \frac{\sum A_n \bar{y}_n}{A_T}$$

[1 mark]



Can be split into 5 areas:

	Width	Length	$\bar{y}_n$	A_n	$A_n\bar{y}_n$
A1	20	10	5	200	1000
A2	60	20	20	1200	24000
A3	20	30	45	600	27000
A4	80	20	70	1600	112000
A5	20	30	95	600	57000
			Sum:	4200	221000
				$\bar{y}$	**52.6**

[3 marks]

(b)

$$I = \sum \left(\frac{BD^3}{12} + A_n (\bar{y}_n - \bar{y})^2 \right)$$

[2 marks]

$\bar{y}_n - \bar{y}$	$A_n(\bar{y}_n - \bar{y})^2$	$\frac{BD^3}{12}$	$A_n(\bar{y}_n - \bar{y})^2 + \frac{BD^3}{12}$
-47.6	453515	1667	455181
-32.6	1276803	40000	1316803
-7.6	34830	45000	79830
17.4	483356	53333	536689
42.4	1077687	45000	1122687
		I	**1851814**

[3 marks]

(c)

General equation

$$\tau = \frac{SQ}{Iz}$$

[1 mark]

A & E are free surfaces so:

$$\boldsymbol{\tau_A = \tau_E = 0\ \mathrm{MPa}}$$

[1 mark]

Can use area below at B, 2 values due to section change:

$$\tau_{B1} = \frac{48000 \times (10 \times 20) \times (56.2 - 5)}{1851814 \times 20} = \mathbf{12.3\ MPa}$$

[1 mark]

$$\tau_{B2} = \frac{48000 \times (10 \times 20) \times (56.2 - 5)}{1851814 \times 60} = \mathbf{4.1\ MPa}$$

[1 mark]

At C, need to consider the effect of two areas and 2 values:

In this case $Q = \sum A(y_n - y)$ and therefore:

$$\tau_{C1} = \frac{48000 \times \big((10 \times 20) \times (56.2 - 5)\big) + (20 \times 60) \times (56.2 - 20))}{1851814 \times 60} = \mathbf{21.0\ MPa}$$

[1 mark]

$$\tau_{C2} = \frac{48000 \times \big((10 \times 20) \times (56.2 - 5)\big) + (20 \times 60) \times (56.2 - 20))}{1851814 \times 20} = \mathbf{63.0\ MPa}$$

[1 mark]

At G, centroid (using areas above):

$$\tau_G = \frac{48000 \times ((30 \times 20) \times (95 - 56.2) + (30 \times 20) \times (70 - 56.2) + \big((60 - 56.2) \times 20\big) \times \left(\frac{60 - 56.2}{2}\right)}{1851814 \times 20}$$

$$= \mathbf{69.03\ MPa}$$

[2 marks]

At D, 2 values

$$\tau_{D1} = \frac{48000 \times \big((30 \times 20) \times (95 - 56.2)\big) + (20 \times 80) \times (70 - 56.2))}{1851814 \times 20} = \mathbf{69.0\ MPa}$$

[1 mark]

$$\tau_{D2} = \frac{48000 \times \big((30 \times 20) \times (95 - 56.2)\big) + (20 \times 80) \times (70 - 56.2))}{1851814 \times 80} = \mathbf{17.25\ MPa}$$

[1 mark]

At E, 2 values

$$\tau_{E1} = \frac{48000 \times (10 \times 20) \times (95 - 56.2)}{1851814 \times 80} = \mathbf{8.24\ MPa}$$

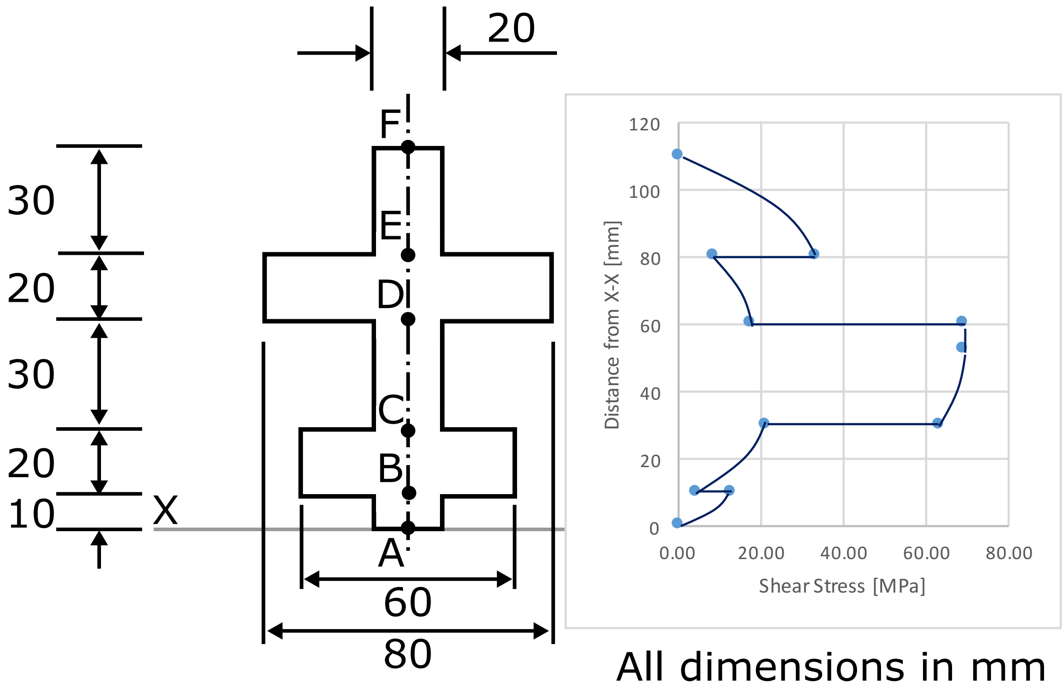
[1 mark]

$$\tau_{E2} = \frac{48000 \times (10 \times 20) \times (95 - 56.2)}{1851814 \times 20} = \mathbf{32.96\ MPa}$$

[1 mark]

(d)

Location	Distance from X-X [mm]	Shear stress [MPa]
A	**0**	**0.00**
B	**10**	**12.34**
B	**10**	**4.11**
C	**30**	**21.02**
C	**30**	**63.07**
G	**52.6**	**69.03**
D	**60**	**69.00**
D	**60**	**17.25**
E	**80**	**8.24**
E	**80**	**32.96**
F	**110**	**0.00**



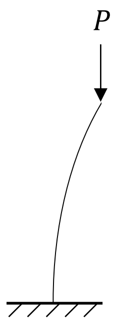
All dimensions in mm

[4 marks]

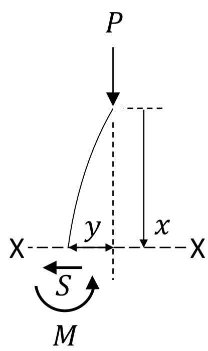
5.

(a)

Below is a diagrammatic representation of a fixed-free strut.



Sectioning this beam in order to determine the bending moment:



[1 mark]

Taking moments about the section position, X-X:

$$M = Py \qquad (1)$$

[1 mark]

2nd order differential equation for a beam under bending:

$$EI\frac{d^2y}{dx^2} = M$$

Substituting (1) into this:

$$EI\frac{d^2y}{dx^2} = Py$$

$$\therefore \frac{d^2y}{dx^2} - \frac{Py}{EI} = 0 \qquad (2)$$

If,

$$\alpha^2 = \frac{P}{EI} \qquad (3)$$

Then substituting this into (2) gives:

$$\frac{d^2y}{dx^2} - \alpha^2 y = 0$$

[2 marks]

Then letting $y = A_O e^{\alpha x}$, we get:

$$y = A sin(\alpha x) + B cos(\alpha x) \qquad (4)$$

where A_O, A and B are constants.

Boundary conditions:

(BC1) At $x = 0$, $y = 0$, therefore from (4):

$$B = 0$$

[1 mark]

(BC2) At $x = L$, $y = 0$, therefore from (4):

$$A\alpha \cos \alpha L = 0$$

[1 mark]

Since $A \neq 0$ for non-trivial solution, i.e. $\cos \alpha L = 0$:

$$\alpha L = \frac{n\pi}{2}$$

[1 mark]

Substituting (3) into this:

$$\sqrt{\frac{P}{EI}} L = \frac{n\pi}{2}$$

$$\therefore \boldsymbol{P} = \frac{\boldsymbol{n^2\pi^2EI}}{\boldsymbol{4L^2}}$$

where $n = 1, 2, \ldots$

[1 mark]

(b)

(i) at $x = 0$, $y = 0$ & $\frac{dy}{dx} = 0$;

[2 marks]

and at $x = L$, $y = 0$, $\frac{dy}{dx} = 0$

[2 marks]

(ii) at $x = 0$, $y = 0$;

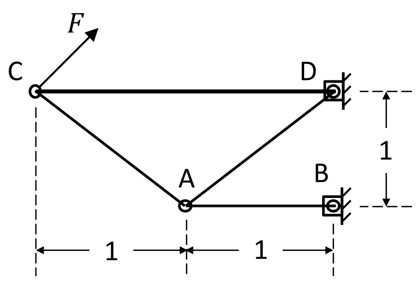
[1 mark]

and at $x = L$:, $y = 0$

[1 mark]

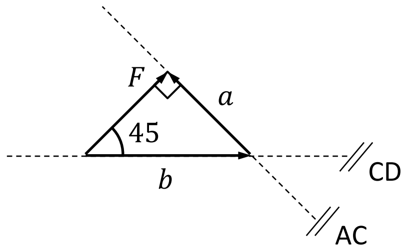
(c)

(i)



Section BD has been removed from the diagram as there is no force in this member.

Forces are in equilibrium; therefore, force polygons can be used. Drawing a force polygon for the force applied at C:



[2 marks]

From geometry, it can be seen that this is an equilateral triangle. Therefore,

$$a = F$$

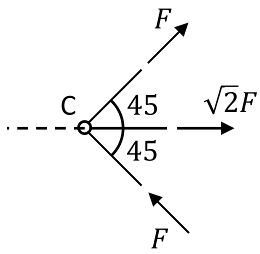
and resolving onto CD:

$$\cos 45 = \frac{F}{b}$$

$$\therefore b = \frac{F}{\cos 45} = \sqrt{2}F$$

[1 mark]

Using the results from the force polygon to draw a free body diagram at joint C:



[1 mark]

Critical load in member AC:

$$P_c{}^{AC} = \frac{\pi^2 EI}{L_{AC}{}^2} = F$$

$$\therefore F = \frac{\pi^2 EI}{L_{AC}{}^2} = \frac{\pi^2 EI}{\left(\sqrt{2}\right)^2} = \frac{\pi^2 EI}{2}$$

[1 mark]

Critical load in member CD:

$$P_c{}^{CD} = \frac{\pi^2 EI}{L_{CD}{}^2} = \sqrt{2}F$$

$$\therefore F = \frac{\pi^2 EI}{\sqrt{2}{L_{CD}}^2} = \frac{\pi^2 EI}{\sqrt{2} \times 2^2} = \frac{\pi^2 EI}{4\sqrt{2}}$$

[1 mark]

Free body diagram at joint A:

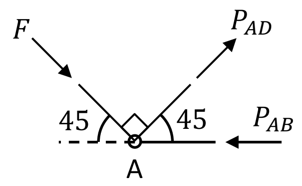


Fig. 1

[1 mark]

Vertical equilibrium gives:

$$F sin(45) = P_{AD} sin(45)$$

$$\therefore P_{AD} = F \qquad (5)$$

Horizontal equilibrium gives:

$$P_{AB} = F cos(45) + P_{AD} cos(45)$$

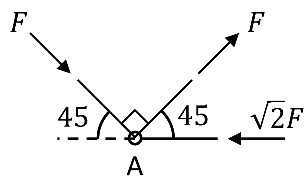
Substituting (5) into this gives:

$$P_{AB} = F cos(45) + F cos(45)$$

$$\therefore P_{AB} = \sqrt{2} F$$

[1 mark]

Fig. 1 can therefore be redrawn as:



Critical load in member AB:

$$P_c{}^{AB} = \frac{\pi^2 EI}{L_{AB}{}^2} = \sqrt{2}F$$

$$\therefore F = \frac{\pi^2 EI}{\sqrt{2}L_{CD}{}^2} = \frac{\pi^2 EI}{\sqrt{2}\times 2^2} = \frac{\pi^2 EI}{4\sqrt{2}}$$

[1 mark]

Critical load in member AD:

$$P_c{}^{AD} = \frac{\pi^2 EI}{L_{AD}{}^2} = F$$

$$\therefore F = \frac{\pi^2 EI}{L_{AC}{}^2} = \frac{\pi^2 EI}{\left(\sqrt{2}\right)^2} = \frac{\pi^2 EI}{2}$$

[1 mark]

Therefore, since members AB and AC are in tension, these will not fail due to buckling. Of the other two members, which are both under compression and therefore potentially subject to buckling, **member CD requires the smallest force to cause buckling and so is the most critical member** due to buckling.

[1 mark]

6.

(a)

Figure Q6.1 shows an element of a straight beam, length δs, which bends to curvature R, due to an applied bending moment M. The angle subtended by the element of beam is $\delta\phi$, also equal to the change in slope of the beam over δs.

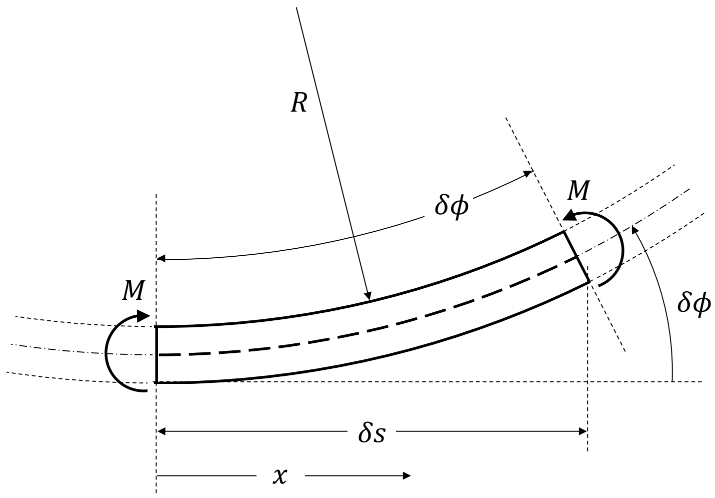


Fig Q6.1 Element of a Beam

[1 mark]

Therefore, the strain energy (work done) for the element, δU, is given by (area under the curve in Fig Q6.2):

$$\delta U = \frac{1}{2} M\delta\phi \qquad (1)$$

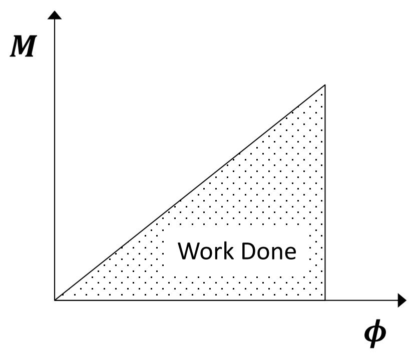


Fig Q6.2 Plot of Strain Energy in a Beam

[1 mark]

Equation of an arc:

$$\delta s = R\delta\phi \qquad (2)$$

Beam Bending equation:

$$\frac{M}{I} = \frac{E}{R} \qquad (3)$$

[1 mark]

Therefore, rearranging (3) for R and substituting this into (2):

$$\delta s = \frac{EI}{M}\delta\phi$$

$$\therefore \delta\phi = \frac{M}{EI}\delta s$$

Substituting this into (1) gives:

$$\delta U = \frac{M^2}{2EI}\delta s \qquad (4)$$

[1 mark]

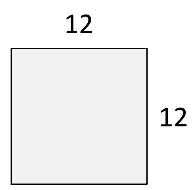
Thus, for a beam of length, L, integrating (4) across this length gives:

$$\boldsymbol{U = \int_0^L \frac{M^2}{2EI}\delta x}$$

[1 mark]

(b)

Second Moment of Area, I, calculation:

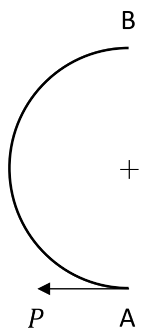


Beam cross-section

$$I = \frac{bd^3}{12} = \frac{12 \times 12^3}{12} = 1{,}728 \text{ mm}^4$$

[2 marks]

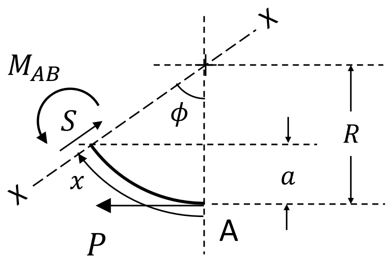
Due to symmetry, half of the ring is considered, as shown in the following figure (ends labelled A and B),



[2 marks]

Section AB (*bending only*):

Free Body Diagram:



[3 marks]

Taking moments about X-X:

$$M_{AB} = Pa = P(R - Rcos\phi) = PR(1 - cos\phi)$$

[2 marks]

Substituting this into the equation for Strain Energy in a beam under bending gives,

$$U_{AB} = \int \frac{{M_{AB}}^2}{2EI} ds = \int_0^{\pi} \frac{\left(PR(1 - cos\phi)\right)^2}{2EI} Rd\phi$$

where,

$$ds = Rd\phi$$

$$\therefore U_{AB} = \frac{P^2R^3}{2EI} \int_0^{\pi} (1 - cos\phi)^2 d\phi = \frac{P^2R^3}{2EI} \int_0^{\pi} (1 - 2cos\phi + cos^2\phi) d\phi$$

[3 marks]

Substituting in the trig identity given in the question,

$$U_{AB} = \frac{P^2R^3}{2EI}\int_0^{\pi}\left(1 - 2cos\phi + \frac{1 + cos2\phi}{2}\right)d\phi = \frac{P^2R^3}{2EI}\int_0^{\pi}\left(\frac{3}{2} - 2cos\phi + \frac{1}{2}cos2\phi\right)d\phi$$

$$= \frac{P^2R^3}{2EI}\left[\frac{3}{2}\phi - 2sin\phi + \frac{1}{4}sin2\phi\right]_0^{\pi}$$

$$\therefore U_{AB} = \frac{3P^2R^3\pi}{4EI} \qquad (5)$$

[2 marks]

Horizontal deflection at position A, u_{v_A}:

Differentiating (5) with respect to applied load, P:

$$u_{h_A} = \frac{\partial U}{\partial P} = \frac{3PR^3\pi}{2EI} = \frac{3 \times 125 \times 200^3 \times \pi}{2 \times 200{,}000 \times 1728}$$

$$\therefore u_{h_A} = 13.64 \text{ mm}$$

[2 marks]

This represents the deflection of the left side of the split. The right side of the split will deflect by the same amount.

[2 marks]

Therefore, **total split opening is $\mathbf{2 \times u_{h_A} = 27.28\ mm}$**

[2 marks]